

Catch Me If You Can!

I. UNIT OVERVIEW & PURPOSE:

The purpose of this unit is to use random acts of disorderly conduct to connect geometry and algebra concepts to real world applications. These concepts include, but are not limited to, distance, equations of the curve of best fit, similar geometric objects in two or three dimensions, right triangle trigonometry, and evaluating functions.

II. UNIT AUTHOR:

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III. COURSE:

Mathematical Modeling: Capstone Course

IV. CONTENT STRAND:

Geometry/Algebra I/Algebra II

V. OBJECTIVES:

- The student will evaluate functions.
- The student will find the equation of the curve of best fit for given data.
- The student will use a table of values to find the equation of best fit.
- The student will use right triangle trigonometry to calculate the measure of angles and sides.
- The student will use the area and circumference formulas for a circle.
- The student will use the Pythagorean Theorem and its converse and properties of special right triangles to solve real-world problems.
- The student will use similar geometric objects in two or three dimensions to solve real-world problems.
- The student will investigate zeros of a function.

VI. MATHEMATICS PERFORMANCE EXPECTATION(s):

MPE. 2: The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE. 5: The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

MPE. 7: The student will use similar geometric objects in two- or three-dimensions to

- a) compare ratios between side lengths, perimeters, areas, and volumes;
- b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
- c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
- d) solve real-world problems about similar geometric objects.

MPE 9: The student will design and conduct an experiment/survey. Key concepts

include

- a) sample size;
- b) sampling technique;
- c) controlling sources of bias and experimental error;
- d) data collection; and
- e) data analysis and reporting.

MPE. 11: The student will use angles, arcs, chords, tangents, and secants to

- a) investigate, verify, and apply properties of circles;
- b) solve real-world problems involving properties of circles; and
- c) find arc lengths and areas of sectors in circles.

MPE. 13: The student will investigate and describe the relationships among solutions of an equation, zeros of a function, x-intercepts of a graph, and factors of a polynomial expression.

MPE 14: The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

VII. CONTENT:

Several acts of mischief have been enacted at your school. You have decided to get on the trail of these culprits. You grab your detective kit and investigate the crimes that have been committed. After studying several crime scenes, you notice that your mathematical skills are very beneficial. Some examples of how you will put those skills to use are but not limited to: Finding out at what angle a damaging rock was thrown at the school, using similar figures to compare a photo of the crime scene to the actual crime scene, determining the area and circumference of the circular cell where the captured criminals will be confined, using quadratic equations to determine if a rocket caused damage, using curve of best fit to determine from where the stink bomb was thrown, and using circumference and drawings to set up a teacher's classroom.

VIII. REFERENCE/RESOURCE MATERIALS:

- Classroom set of graphing calculators
- Graph paper
- Area Formulas
- Classroom set of laptops or iPads for internet access and Sketchpad or GeoGebra
- Microsoft Excel
- Measuring tools

IX. PRIMARY ASSESSMENT STRATEGIES:

- Written paragraph about discussion of the problem
- Calculate the distance from the center to the edge of the yard
- Write equations based on given information
- Evaluate functions
- Determining line of best fit
- Solving real world problems using right triangle trigonometry

X. EVALUATION CRITERIA:

- A rubric will be used to include the main points of the discussion.
- Look for the use of the area and circumference formula to calculate the cell position.
- Check equations for given information.
- Determine if functions are evaluated correctly
- Determine if area has been calculated correctly
- Check to see if the line of best fit has been calculated correctly
- Verify that right triangle trigonometry has been used correctly
- Verify that volume has been calculated correctly

XI. INSTRUCTIONAL TIME:

7 – 90 minute blocks

Lesson 1: It's Raining Rocks!

Strand

Geometry

Mathematical Objective(s)

- The student will use the Pythagorean Theorem to find a missing side given 2 sides of a right triangle.
- The student will use right triangle trig to find a missing side or angle.

Mathematics Performance Expectation(s)

MPE. 5: The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

Related SOL

G.8: The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

NCTM Standards

- Use trigonometric relationships to determine lengths and angle measures.

Materials/Resources

- Classroom set of graphing calculators
- Graph paper

Assumption of Prior Knowledge

- Students should have a mastery of the Pythagorean Theorem.
- Students should have a working knowledge of basic triangle vocabulary – leg, hypotenuse, etc.

Introduction: Setting Up the Mathematical Task

Your first detective job will be to determine at what angle the vandal threw the rock at the school. You are able to measure how tall the school is and how far away from the school that the vandal launched the rock. You are also assigned to investigate another related crime. However, this time, you need to determine how tall the courthouse is.

You know again how far the vandal was from courthouse and you know that he threw the rock at a 50° angle.

- Individual work (20 minutes), small group discussion/work (55 minutes), whole class discussion (15 minutes)
- Student will work in groups of 2-3 on this activity.
- Students should draw an overhead picture of the school and the courthouse.

Student Exploration:

- **Individual Work** – The individual student will label each of the triangle drawings and determine the sides/angles that are given and also those that need to be found.
- **Small Group Work** - The students will reconstruct the crime scene by recreating the scene inside their gymnasium. They will use actual measurements and determine possible answers and those that are impossible due to miscalculations and those that could be wrongly labeled.
- **Whole Class Sharing/Discussion** – The small groups will then come back together to discuss the lengths and angle answers they have found. They will also determine by debate if the vandal could be the same person by heights and angles.

Student/Teacher Actions:

- **What should students be doing?** The students should be calculating the height of the courthouse and the angle at which the rock was thrown at the school. They should be actively working with their groups measuring and determining if their answer is a possibility.
- **What should teachers be doing to facilitate learning?** Teachers should be constantly monitoring the groups to ensure they are using the correct Pythagorean Theorem and correct trig ratios. They should also be available for methodological questions but not hints on where to actually place the given lengths and angles. If students are having trouble, they can use the textbook, internet, or classmates for help.
- **Possible questions** – Possible problems the students may face are those dealing with whether to use the Pythagorean Theorem or trig ratios. Once the students correctly label the triangle, they should be able to discover the correct equation quickly.
- **Technology Integration or Cooperative/Collaborative Learning Possibilities** – Students will be able to create the circle from their original sketches on Sketchpad or using GeoGebra software.

Monitoring Student Responses

- Students will communicate with their peers in a group discussion why they chose to label the triangle the way they did.
- Students will continue to use trial and error if their answer is not logical.

- Teacher and/or students will record those lengths and angles that work and also those that will not work and why.
- Teacher will also extend extra instruction to those struggling and will also re-shuffle the groups so that different ideas can be spread by different students into different groups.
- Summary
 - Students will write a small paragraph, along with their triangle drawing as an illustration, to summarize finding the correct length or angle.
- Students will turn in group work drawing and individual paragraphs to document their work.

Teacher's Note:

- Students will be using the idea of the vandal's crime of throwing the rock to practice their right triangle trigonometry and Pythagorean Theorem skills.
- Teachers will create their own sketches of the courthouse and other window-breaking problems. This will enable to teachers to adapt this lesson to the differentiated learning styles of their individual students.

Assessment

- **Exit Pass will be called "Glad and Sad." This will be handed to the student as a chart template on paper. The student will list what they would be "Glad" to see on a test and what they would be "Sad" to see on a test. This is not only what they did not like but also what they did not understand.**

Some ideas to think about when completing this chart:

- What did you like most about this lesson?
- What did you like least?
- What did you learn?
- What questions do you still have about the material covered in the lesson?

Exit Passes will be evaluated to see what each student learned and if they still have any questions about the material covered that day. If the students still have questions about the material covered, they will be addressed at the beginning of class the next day.

- Homework will be graded as a homework grade. It will determine how the students comprehended the information learned in class.

Homework Questions

- **PART 1 & 2** – Will be included on a Student-Ready Worksheet at the end of the lesson.

Extensions and Connections (for all students)

- Lesson extensions/follow-up
- Connections to content in other subject areas. Examples of these would be included in the “Glad and Sad” portion of each lesson. Students will have to write complete sentences and organize ideas for this activity, therefore, it could be considered as grammatical work in English.

Strategies for Differentiation

The exit pass is developed so that students are able to express themselves by word or by picture. They can list or draw a picture of what they are glad or sad to see in the lesson. Another way the lesson is differentiated is the first part of the homework problems. This is a matching exercise. The students will just have to identify the formulas and know their “owner” rather than writing them out by memory. This is just a way the lesson is differentiated for different types of learners and different learning strategies.

Exit Pass – Homework 1

GLAD



SAD



Homework – Lesson 1

Part 1

Match the correct formula to its owner:

1. Pythagorean Theorem

a) adjacent
hypotenuse

2. Sine

b) opposite
adjacent

3. Cosine

c) $a^2 + b^2 = c^2$

4. Tangent

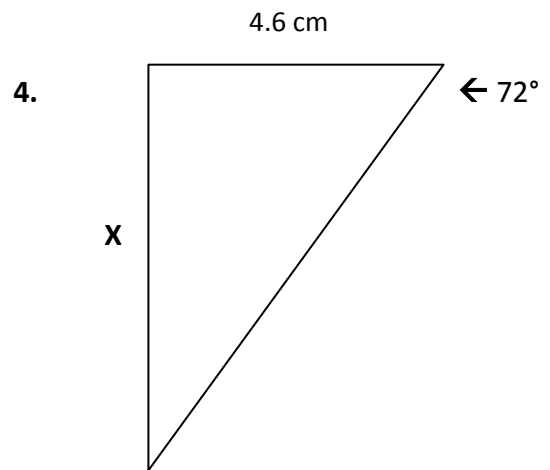
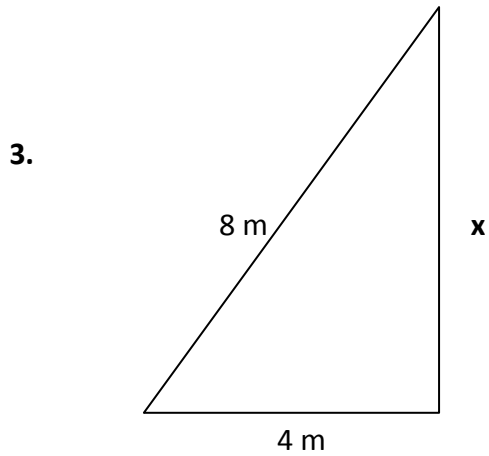
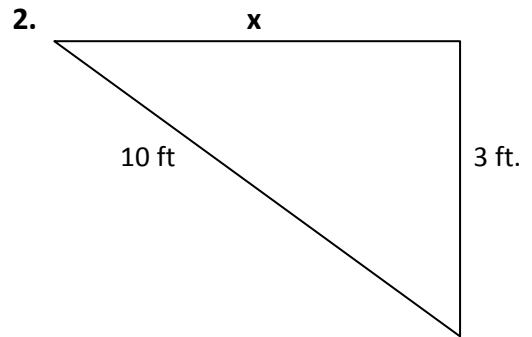
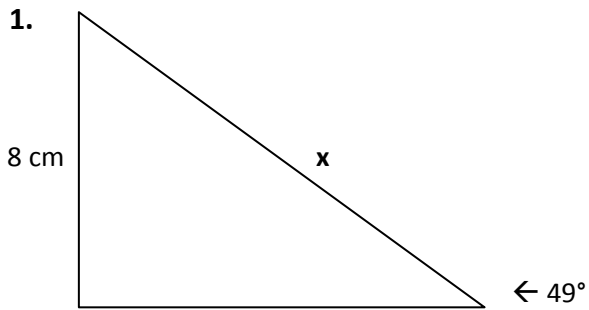
d) opposite
hypotenuse

Homework – Lesson 1

Part 2

This worksheet is created to help students practice scenarios similar to the crime scene.

Given the right triangle, find the missing angle or side length. Round to the nearest tenth.



Keys for Homework 1

Part 1

Match the correct formula to its owner:

- | | | |
|------------------------|---|----------------------------------|
| 1. Pythagorean Theorem | → | a) <u>adjacent</u>
hypotenuse |
| 2. Sine | → | b) <u>opposite</u>
adjacent |
| 3. Cosine | → | c) $a^2 + b^2 = c^2$ |
| 4. Tangent | → | d) <u>opposite</u>
hypotenuse |

Keys for Homework 1 - Part 2

1. 10.6 cm.

2. 9.5 ft.

3. 6.9 m

4. 14.2 cm

Lesson 2: Comparing Crime Scenes

Strand

Geometry

Mathematical Objective(s)

- The student will use similar geometric objects in two- or three-dimensions to compare ratios between side lengths, perimeters, areas, and volumes.
- The student will determine how changes in one or more dimensions of an object affect area and/or volume of the object
- The student will solve real-world problems about similar geometric objects.

Mathematics Performance Expectation(s)

- MPE. 7: The student will use similar geometric objects in two- or three-dimensions to
- a) compare ratios between side lengths, perimeters, areas, and volumes;
 - b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
 - c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
 - d) solve real-world problems about similar geometric objects.

Related SOL

- G.7: The student, given information in the form of a figure or statement, will prove two triangles are similar, using algebraic and coordinate methods as well as deductive proofs.

NCTM Standards

- Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them.

Materials/Resources

- Classroom set of graphing calculators
- Graph paper

Assumption of Prior Knowledge

- Students should have a mastery of the Pythagorean Theorem.
- Students should have a working knowledge of basic circle vocabulary – leg, hypotenuse, etc.

Introduction: Setting Up the Mathematical Task

Your second detective job will be to determine if the sketch drawing of the crime scene is similar to the actual drawing of the crime scene. You will have to determine if the sides are proportional and find the scale factor.

- Individual work (25 minutes), small group discussion/work (50 minutes), whole class discussion (15 minutes)
- Student will work in groups of 2-3 on this activity.
- Students should draw a picture of the crime scene and take the actual measurements of the crime scene.

Student Exploration:

- **Individual Work** – The individual student will draw and label each of the similar triangle drawings and determine if the triangles are similar.
- **Small Group Work** - The students will measure a recreated crime scene that the teacher has set up in the school's gymnasium. The groups will then have to take their drawing of the crime scene and then perform similarity tests to see if the two sets of measurements are similar.
- **Whole Class Sharing/Discussion** – The small groups will then come back together to discuss the scale factor answers they have found. They will also determine by debate if they could submit and use their drawings on an actual case.

Student/Teacher Actions:

- **What should students be doing?** The students should be completing their drawings of the crime scene and then actually measuring the recreated scene in small groups. They should be actively working with their groups measuring and determining if their scale factors are a possibility.
- **What should teachers be doing to facilitate learning?** Teachers should be constantly monitoring the groups to ensure they are using correct similarity strategies. They should also be available for methodological questions but not hints on where to actually place the lengths and angles. If students are having trouble, they can use the textbook, internet, or classmates for help.
- **Possible questions** – Possible problems the students may face are those dealing with how to find the correct scale factor and labeling the sides correctly. They may need help setting up ratios.
- **Technology Integration or Cooperative/Collaborative Learning Possibilities** – Students will be able to create the circle from their original sketches on Sketchpad or using GeoGebra software.

Monitoring Student Responses

- Students will communicate with their peers in a group discussion why they chose to label the triangle the way they did.
- Students will continue to use trial and error if their answer is not logical.
- Teacher and/or students will record those scale factors that work and also those that will not work and why.
- Teacher will also extend extra instruction to those struggling and will also re-shuffle the groups so that different ideas can be spread by different students into different groups.
- Summary
 - Students will write a small paragraph, along with their triangle drawings as an illustration, to summarize finding the correct scale factor.
 - Students will turn in group work drawing and individual paragraphs to document their work.

Assessment

- **Exit Pass will be called “Glad and Sad.” This will be handed to the student as a chart template on paper. The student will list what they would be “Glad” to see on a test and what they would be “Sad” to see on a test. This is not only what they did not like but also what they did not understand.**

Some ideas to think about when completing this chart:

- What did you like most about this lesson?
- What did you like least?
- What did you learn?
- What questions do you still have about the material covered in the lesson?

Homework Questions

PART 1– Will be included on a Student-Ready Worksheet at the end of the lesson.

PART 2 – The students will be asked to complete a similar figures drawing. They will use the drawing they constructed of the crime scene and then of the recreated crime scene in the gym with real-life measurements. A rubric will be provided for grading.

- **Teacher’s Note** - Students will be using the idea of the recreated fictitious crime scene from the school gym to help the students practice finding scale factors. This can be recreated with string, jump ropes, anything that can be connected to form the polygon. If necessary, this lesson can be separated into two lessons for two days. The

first day could consist of the background and practice of similar figures and the second day would include the activity in the gym.

Exit Passes will be evaluated to see what each student learned and if they still have any questions about the material covered that day. If the students still have questions about the material covered, they will be addressed at the beginning of class the next day.

- Homework will be graded as a homework grade. It will determine how the students comprehended the information learned in class.

Extensions and Connections (for all students)

- Lesson extensions/follow-up
- Connections to content in other subject areas. Examples of these would be included in the “Glad and Sad” portion of each lesson. Students will have to write complete sentences and organize ideas for this activity, therefore, it could be considered as grammatical work in English.

Strategies for Differentiation

The exit pass is developed so that students are able to express themselves by word or by picture. They can list or draw a picture of what they are glad or sad to see in the lesson. Another way the lesson is differentiated is the first part of the homework problems. This is a matching exercise. The students will just have to identify the formulas and know their “owner” rather than writing them out by memory. This is just a way the lesson is differentiated for different types of learners and different learning strategies.

Exit Pass – Homework 2

GLAD



SAD

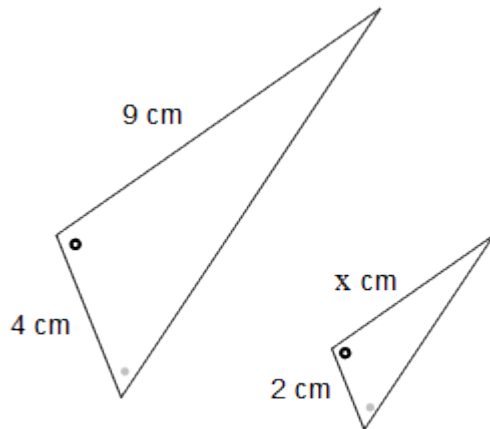


Homework 2- Part 1

This worksheet was created to help simulate the recreated crime scene for the students and to practice the skill of finding scale factors.

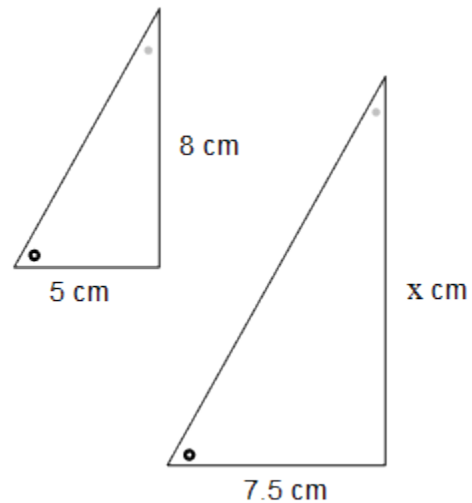
Write the **scale factors** and **values for x** for the pairs of similar triangles below.

1.



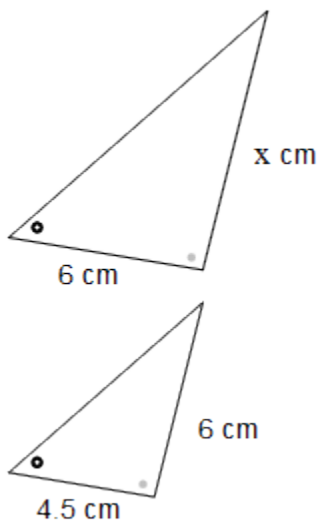
Scale Factor: ____ $x =$ ____

2.

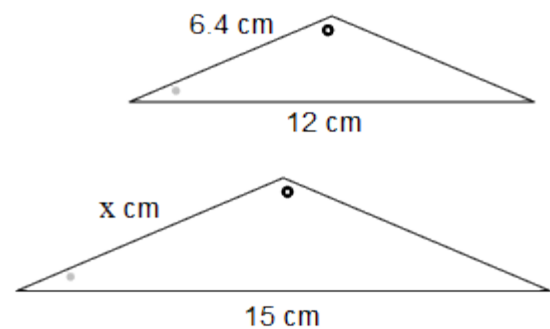


Scale Factor: ____ $x =$ ____

3.

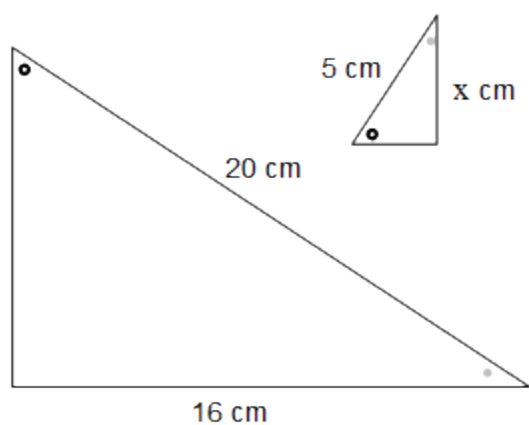


Scale Factor: ____ $x =$ ____

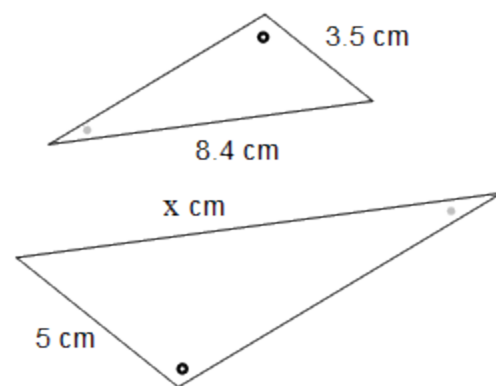


4.

Scale Factor: ____ $x =$ ____



5. Scale Factor: _____ \times = _____



6. Scale Factor: _____ \times = _____

Homework 2 – Part 1

Answer Key

1. $x = 4.5$

Scale Factor = $\frac{9}{4}$

2. $x = 12$

Scale Factor = $\frac{8}{5}$

3. $x = 8$

Scale Factor = $\frac{3}{4}$

4. $x = 8$

Scale Factor = $\frac{8}{15}$

5. $x = 4$

Scale Factor = $\frac{5}{4}$

6. $x = 12$

Scale Factor = $\frac{7}{10}$

Rubric for Part 2-Homework 2

	Provides Clear Understanding	Needs Improvement	Fails to Understand
Identifying the appropriate dimensions			
Using the correct ratios			
Finding scale factors that will work in real-life situations.			

Lesson 3: We're Gonna Find You!

Strand

Geometry

Mathematical Objective(s)

- The student will use angles, arcs, chords, tangents, and secants to investigate, verify, and apply properties of circles;
- The student will solve real-world problems involving properties of circles
- The student will find arc lengths and areas of sectors in circles.

Mathematics Performance Expectation(s)

- MPE. 11: The student will use angles, arcs, chords, tangents, and secants to
- a) investigate, verify, and apply properties of circles;
 - b) solve real-world problems involving properties of circles; and
 - c) find arc lengths and areas of sectors in circles.

Related SOL

- G.11: The student will use angles, arcs, chords, tangents, and secants to
- a) investigate, verify, and apply properties of circles;
 - b) solve real-world problems involving properties of circles; and
 - c) find arc lengths and areas of sectors in circles.

NCTM Standards

- The student will understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.

Materials/Resources

- Classroom set of graphing calculators
- Graph paper

Assumption of Prior Knowledge

- Students should have a mastery of the area, circumference, sector area, and arc length.
- Students should have a working knowledge of basic circle vocabulary – diameter, radius, etc.

Introduction: Setting Up the Mathematical Task

You are hot on the vandals' trail and in order to deviate from your routine, you decide to begin planning on where to keep the criminals once they are apprehended.

You have decided the amount of land you have is best suited for circular cells. However, you must decide 1) how many cells you can fit in the allotment of land, 2) how much area each cell will cover, and 3) what the circumference of each cell is.

- Individual work (15 minutes), small group discussion/work (60 minutes), whole class discussion (15 minutes)
- Student will work in groups of 2-3 on this activity.
- Students should draw a picture of land that contains the maximum number of cells with correct areas and circumferences.

Student Exploration:

- **Individual Work** – The individual student will draw and label each of the circular cells.
- **Small Group Work** - The students will label the radius and determine central angles if necessary. The groups will also work on finding the areas and circumferences together. Then, they should formulate a drawing that contains the maximum number of cells.
- **Whole Class Sharing/Discussion** – The small groups will then come back together to discuss which group had the most efficient plan with the correct areas and circumferences.

Student/Teacher Actions:

- **What should students be doing?** The students should be completing their drawings of the circular cells and deciding how many will fit in the allotment of land. They should be actively working with their groups measuring and determining if their drawing will possibly work.
- **What should teachers be doing to facilitate learning?** Teachers should be constantly monitoring the groups to ensure they are using correct circle formulas. They should also be available for methodological questions but not hints on where to actually place the radii. If students are having trouble, they can use the textbook, internet, or classmates for help.
- **Possible questions** – Possible problems the students may face are those dealing with how to find the radius from a diameter and labeling the circle correctly. They may also need help determining who or why a certain group had the most efficient drawing.
- **Technology Integration or Cooperative/Collaborative Learning Possibilities** – Students will be able to create the circle from their original sketches on Sketchpad or using GeoGebra software.

Monitoring Student Responses

- Students will communicate with their peers in a group discussion why they chose to set up their drawing the way they did.
- Students will continue to use trial and error if their answer is not logical.

- Teacher and/or students will record those scale factors that work and also those that will not work and why.
- Teacher will also extend extra instruction to those struggling and will also re-shuffle the groups so that different ideas can be spread by different students into different groups.
- Summary
 - Students will write a small paragraph, along with their circular drawings as an illustration, to summarize why their circular cell drawing would be most efficient.
 - Students will turn in group work drawing and individual paragraphs to document their work.

Assessment

- **Exit Pass will be called “Glad and Sad.” This will be handed to the student as a chart template on paper. The student will list what they would be “Glad” to see on a test and what they would be “Sad” to see on a test. This is not only what they did not like but also what they did not understand.**

Some ideas to think about when completing this chart:

- What did you like most about this lesson?
- What did you like least?
- What did you learn?
- What questions do you still have about the material covered in the lesson?

Homework Questions

PART 1– Will be included on a Student-Ready Worksheet at the end of the lesson.

PART 2 – The students will be asked to complete a circular cell drawing. They will use 8.5”x11” paper to show the most efficient way to have the most cells. They should also choose a radius that works. A rubric will be provided for grading.

Exit Passes will be evaluated to see what each student learned and if they still have any questions about the material covered that day. If the students still have questions about the material covered, they will be addressed at the beginning of class the next day.

- Homework will be graded as a homework grade. It will determine how the students comprehended the information learned in class.

Extensions and Connections (for all students)

- Lesson extensions/follow-up
- Connections to content in other subject areas. Examples of these would be included in the “Glad and Sad” portion of each lesson. Students will have to write complete sentences and organize ideas for this activity, therefore, it could be considered as grammatical work in English.

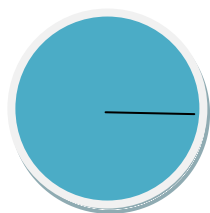
Strategies for Differentiation

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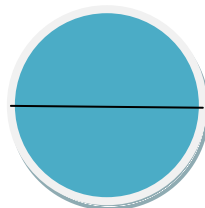
Homework 3-Part 1

Find the area and circumference of the following circles.
Round to the nearest tenth.

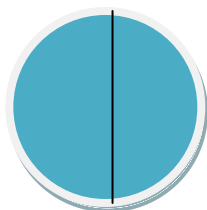
1. Radius = 3 cm



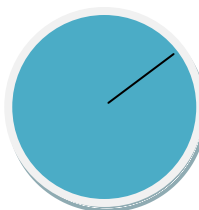
2. Diameter = 10 ft



3. Diameter = 8 in



4. Radius = 7 m



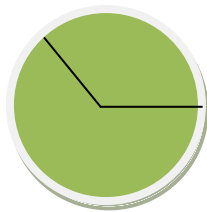
Homework 3 – Part 2

Find the sector area of problems 1-3.

Find the arc length of problems 4-6.

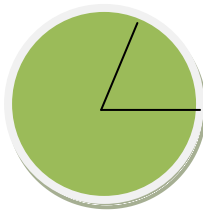
Round to the nearest tenth.

1.



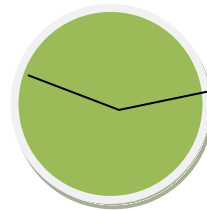
central angle = 110°
radius = 10 cm

2.



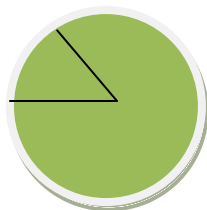
central angle = 65°
radius = 4 cm

3.



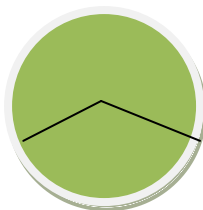
central angle = 135°
radius = 6 cm

4.



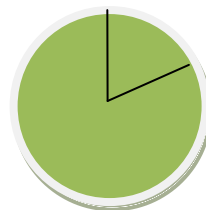
central angle = 50°
radius = 9 cm

5.



central angle = 100°
radius = 7 cm

6.



central angle = 35°
radius = 3 cm

Key for Homework 3-Part 1

1. $A = 28.3 \text{ cm}^2$

$C = 18.8 \text{ cm}$

2. $A = 78.5 \text{ ft}^2$

$C = 31.4 \text{ ft}$

3. $A = 50.3 \text{ in}^2$

$C = 25.1 \text{ in}$

4. $A = 153.9 \text{ m}^2$

$C = 44.0 \text{ m}$

Key for Homework 3-Part 2

1. $SA = 24.0 \text{ cm}^2$

2. $SA = 9.1 \text{ cm}^2$

3. $SA = 42.1 \text{ cm}^2$

4. $AL = 7.9 \text{ cm}$

5. $AL = 12.2 \text{ cm}$

6. $AL = 18 \text{ cm}$

Rubric for Homework 3-Part 3

	Provides Clear Understanding	Needs Improvement	Fails to Understand
Identifying the appropriate parts of a circle			
Using the correct formulas			
Finding possible cell layouts that will work in real-life situations.			

Lesson 4: Whose Rocket Was It?

Strand

Algebra I / Algebra II

Mathematical Objective(s)

- The student will use quadratic equations to solve real world problems.
- The student will use the formula for parabolic projectiles to determine landing of a rocket.
- The student will use GeoGebra to graph their quadratic equation to determine if the physics teacher is guilty.

Mathematics Performance Expectation(s)

MPE 14: The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

Related SOL

- A.4 The student will solve multistep linear and quadratic equations in two variables, including
- c) solving quadratic equations algebraically and graphically;
 - f) solving real-world problems involving equations and systems of equations.
- Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

NCTM Standards

- identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
- draw reasonable conclusions about a situation being modeled.
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.

Materials/Resources

- Classroom set iPads or laptops with GeoGebra
- Paper and Pencil
- Data from Physics Teacher and Crime Scene

Assumption of Prior Knowledge

- Students should have a mastery of quadratic equations.

- Students should have a working knowledge of the parabolic projectile equation.
- Students should have a working knowledge of graphing equations in GeoGebra.

Introduction: Setting Up the Mathematical Task

The Physics class decided last week to launch rockets that they had built in class from the window of the classroom. The day for the launch had perfect weather: sunny, perfect moderate temperatures and no wind. With these conditions, the students would each achieve a perfect flight. Yesterday, a woman from the neighborhood behind the school complained to the principal that she had returned from a trip to find a rocket had knocked over her rose arbor and burned some of her roses. She is demanding the school replace her arbor and roses. The Physics teacher claims that his students' rockets couldn't possibly have gone that far, and they are not to blame. Your job is to determine who is telling the truth. The principal has gathered the following information: the distance from the launch site to the woman's rose arbor and the initial velocity of each students' rocket and the starting height of the classroom window. (Teachers should supply this information to the students.) Teachers can also state different guidelines for determining guilt, such as: if no rockets reached the arbor, all are innocent, if 1-3 students reached the arbor, they are individually guilty and the school is innocent, and if more than 3 students' rockets reached the arbor, the school is responsible.

- Small group discussion/work (70 minutes), whole class discussion (20 minutes)
- Student will work in groups of 2-3 on this activity.

Student Exploration:

- **Small Group Work** – The students in each group will use the information provided to write a quadratic equation for the flight of each rocket. Once this is complete, each group will go to a computer and use GeoGebra to graph all of the equations on a coordinate plane. After plotting the equations, the students should plot the rose arbor as a point on the x-axis with the launch site being on the y-axis. Students will probably want to graph each equation in a different color.
- **Whole Class Sharing/Discussion** – The small groups will then come back together to discuss what they found when the graphs were complete. After discussing any differences in graphs or equations, the class as a whole should determine if any of the students' rocket(s) reached the arbor. If so, they should identify the student(s) responsible.

Student/Teacher Actions:

- **What should students be doing?** The students should be writing equations, graphing them and then analyzing the graphs to determine who is at fault. Students will probably want to graph each equation in a different color.
- **What should teachers be doing to facilitate learning?** The teacher should provide the initial information needed for the problem. They should also provide guidelines for determining guilt. Teachers should be constantly monitoring the groups to ensure they are using the correct equation and that the students are properly graphing the equations written. They should also be available for methodological questions but not hints on how to analyze the graphs. If students are having trouble, they can use the textbook, internet, or classmates for help.
- **Possible questions** – Possible problems the students may face are those dealing with not knowing the proper equation to use, plugging information in incorrectly, not being familiar with graphing in GeoGebra, or not knowing how to plot the parabola on the graph.
- **Technology Integration or Cooperative/Collaborative Learning Possibilities** – Students will use GeoGebra to graph their quadratic equations to allow them to see the graphs at once. Students will probably want to graph each equation in a different color.

Monitoring Student Responses

- Students will communicate with their peers in a group discussion why they wrote the equation the way they did.
- Students will graph to make sure their equation is a parabola and makes sense.
- Teacher and/or students will record the equations for each students' rocket and its landing distance.
- Teacher will also extend extra instruction to those struggling and will also re-shuffle the groups so that different ideas can be spread by different students into different groups.
- **Summary**
 - Students will write a small paragraph, along with a printout of the GeoGebra graph, detailing their findings on the guilt or innocence of the each student.
 - Students will turn in group printout and individual paragraphs to document their work.

Assessment

- **Exit Pass will be called "Glad and Sad."** This will be handed to the student as a chart template on paper. The student will list what they would be "Glad" to see on a test and what they would be "Sad" to see on a test. This is not only what they did not like but also what they did not understand.

Some ideas to think about when completing this chart:

- What did you like most about this lesson?
- What did you like least?
- What did you learn?
- What questions do you still have about the material covered in the lesson?

Homework Questions

PART 1– Will be included on a Student-Ready Worksheet at the end of the lesson.

Exit Passes will be evaluated to see what each student learned and if they still have any questions about the material covered that day. If the students still have questions about the material covered, they will be addressed at the beginning of class the next day.

- Homework will be graded as a homework grade. It will determine how the students comprehended the information learned in class.

Extensions and Connections (for all students)

- Lesson extensions/follow-up
- Connections to content in other subject areas. Examples of these would be included in the “Glad and Sad” portion of each lesson. Students will have to write complete sentences and organize ideas for this activity, therefore, it could be considered as grammatical work in English.

Strategies for Differentiation

The exit pass is developed so that students are able to express themselves by word or by picture. They can list or draw a picture of what they are glad or sad to see in the lesson. Another differentiation strategy would be to have a smaller number of students’ rockets to follow for those who may struggle. Gifted/ accelerated students can have extra parameters added to the data. For example, the teacher could add a maximum flight time based on fuel that would change whether the rocket lands or just runs out of fuel and drops down.

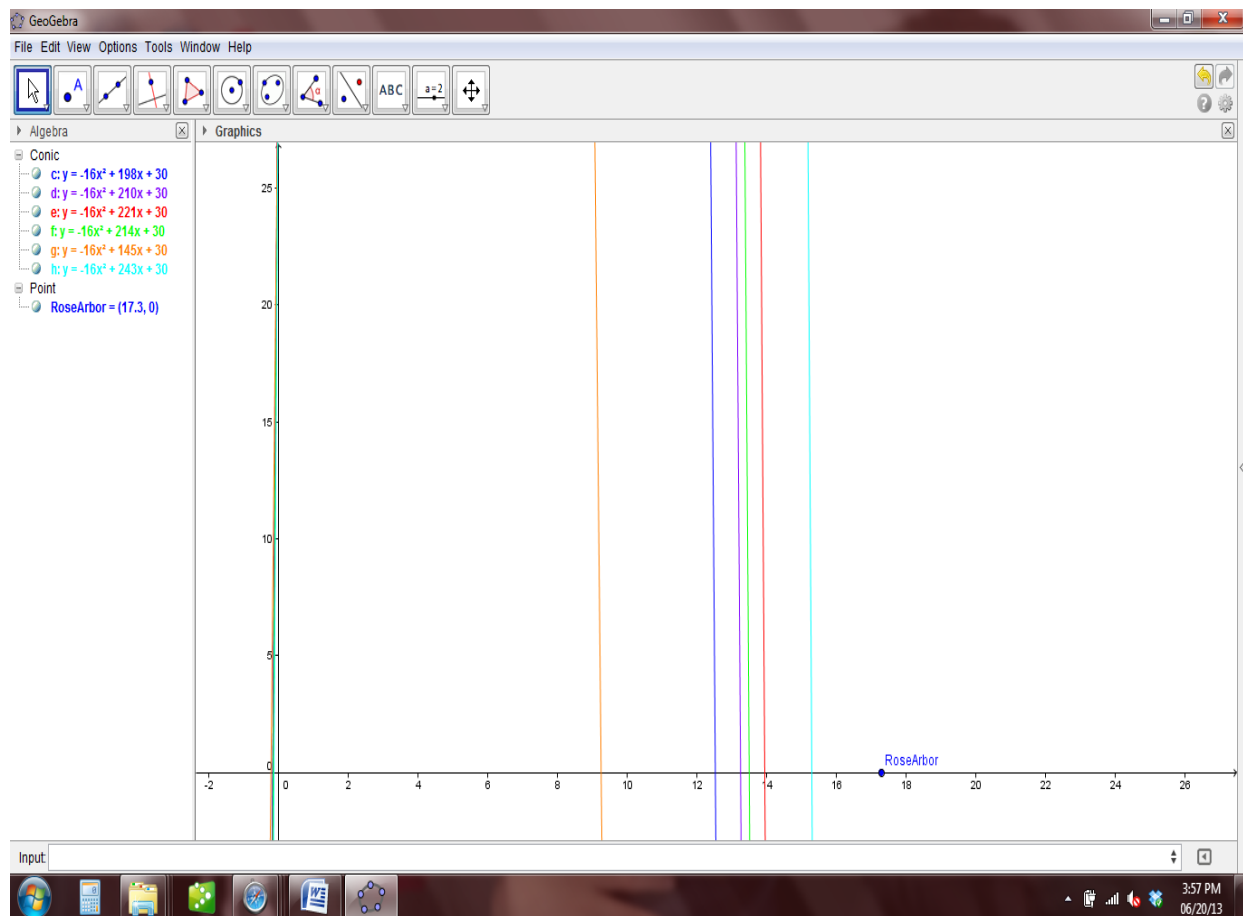
Exit Pass – Homework 4

GLAD



SAD





Here is a screenshot of a hypothetical student project in GeoGebra. This is to use as a reference for the teacher and students on how to do their problems.

Homework: Lesson 4

I. Write the quadratic equation for the given information.

1) Initial velocity = .3 ft/s initial height = 4.6 ft

$f(x) =$ _____

2) Initial velocity = 142 ft/s initial height = -8.3 ft

$f(x) =$ _____

3) Initial velocity = 23.7 ft/s initial height = 14.8 ft

$f(x) =$ _____

II. Graph each above equation on the graphing calculator and find distance when it hits the ground.

1) _____

2) _____

3) _____

ANSWERS

Homework: Lesson 4

III. Write the quadratic equation for the given information.

4) Initial velocity = .3 ft/s initial height = 4.6 ft

$$f(x) = \underline{-16x^2 + .3x + 4.6}$$

5) Initial velocity = 142 ft/s initial height = -8.3 ft

$$f(x) = \underline{-16x^2 + 142x - 8.3}$$

6) Initial velocity = 23.7 ft/s initial height = 14.8 ft

$$f(x) = \underline{-16x^2 + 23.7x + 14.8}$$

IV. Graph each above equation on the graphing calculator and find distance when it hits the ground.

4) .55 ft

5) 8.82 ft

6) 1.95 ft

Lesson 5: Who Stinks?

Strand

Algebra I / Algebra II

Mathematical Objective(s)

- The student will use the scientific method to develop an experiment to solve the problem.
- The student will use quadratic equations to solve real world problems.
- The student will use quadratic regression to determine best fit of data collected.
- The student will use Excel to calculate regression of each of the three sets of data collected.

Mathematics Performance Expectation(s)

MPE 9: The student will design and conduct an experiment/survey. Key concepts include

- a) sample size;
- b) sampling technique;
- c) controlling sources of bias and experimental error;
- d) data collection; and
- e) data analysis and reporting.

MPE 2: The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models.

Mathematical models will include polynomial, exponential, and logarithmic functions.

MPE 14: The student will recognize the general shape of function (absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic) families and will convert between graphic and symbolic forms of functions. A transformational approach to graphing will be employed. Graphing calculators will be used as a tool to investigate the shapes and behaviors of these functions.

Related SOL

A.4 The student will solve multistep linear and quadratic equations in two variables, including

- c) solving quadratic equations algebraically and graphically;
- f) solving real-world problems involving equations and systems of equations.

Graphing calculators will be used both as a primary tool in solving problems and to verify algebraic solutions.

A.11 The student will collect and analyze data, determine the equation of the curve of best fit in order to make predictions, and solve real-world problems, using mathematical models. Mathematical models will include linear and quadratic functions.

NCTM Standards

- Use mathematical models to represent and understand quantitative relationships
- identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
- draw reasonable conclusions about a situation being modeled.
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.

Materials/Resources

- Classroom set iPads or laptops with Excel
- Graphing Calculator
- Paper and Pencil
- Measuring tape or yard stick
- Multistory building or area to simulate a multistory building
- Objects to throw in experiment. (You may want to use objects that won't bounce or make a mark when they hit, so the student can measure the distance out that the object went.)

Assumption of Prior Knowledge

- Students should have a mastery of quadratic equations.
- Students should have a mastery of regression.
- Students should have a working knowledge of graphing calculators.
- Students should have a working knowledge of calculating regression in Excel, (or the teacher may want to do a small tutorial session for the class.)
- Students should be able to set up an experiment that would accurately answer the question.

Introduction: Setting Up the Mathematical Task

Eewwww! Someone threw a stink bomb in the lobby of your school. The lobby is an atrium style area that has balconies on each of the three levels. A teacher took note of the students on each level at the time but turned her back for just a second when the stink bomb was thrown. Your job is to determine what level the stink bomb was thrown from based on the distance from the balcony area the stink bomb landed. You will need to set up an experiment to answer this problem. This information will narrow the suspects down so the principal doesn't have to look at too many students. The principal has gathered the following information: the distance from the base of the balcony area to the stink bomb. (Teachers should supply this information to the students.) Students can be given the height of each balcony or can calculate the height of each balcony using trig ratios for use in the experiment. This will need to be done in multiple stages.

Teacher's Note: The story for this lesson was created by imagining a school setup that is very different from the reality of most school buildings. If you would like to keep with the scenario applying to your school, you will need to adapt the story to fit your school. Having different heights is the ideal scenario. If you do not have a multistory school, you may try using bleachers in the gym and trying to find which row the stink bomb most likely came from. You could also try having students stand or sit at different levels in the classroom to simulate height differences. Another possible scenario would be to use different distances to allow for regression points. The object would be to determine the most likely distance for the thrower to be from the landing site. The scenario given is a guide to help set up the experiment that works best for your class.

- Stage 1: Individual work (10 minutes), Small group discussion/work (20 minutes), whole class discussion (20 minutes)
- Stage 2: Small group discussion/work (80 minutes), whole class discussion (10 minutes)
- Stage 3: Individual work (10 minutes), Small group discussion/work (20 minutes), whole class discussion (10 minutes)
- Student will work in groups of 2-3 on this activity.

Student Exploration:

Stage 1

- **Individual Work** – Each student should brainstorm to come up with an experiment that will allow the students to determine which balcony the stink bomb was thrown.
- **Small Group Work** – The students will come together in groups to discuss the individual brainstorming and come up with a group idea stating the best way to conduct this experiment.
- **Whole Class Sharing/Discussion** – The small groups will then come back together to discuss what they came up with as an experiment. The teacher should lead a group discussion on the good parts and bad parts for each experiment. Through group discussion and teacher input, the students should come to the conclusion that they need to throw several objects off of each balcony and measure the distance out from the balconies each one falls. After gathering data, they will then calculate a regression to use to solve the problem.

Student/Teacher Actions:

- **What should students be doing?** The students should be coming up with an experiment to solve the problem.
- **What should teachers be doing to facilitate learning?** Teachers should be constantly monitoring the groups to ensure they are on the right track. They should also be

available for methodological questions but not hints on specific processes. The teacher should also facilitate classroom discussion to guide students to the correct answer. If students are having trouble, they can use the textbook, internet, or classmates for help.

- **Possible questions** – Possible problems the students may face are getting started on the path for a proper experiment and determining multiple throws are needed.

Monitoring Student Responses

- Students will communicate with their peers in a group discussion their thinking.
- Students will come up with a step-by-step experiment that will provide a correct answer.
- Teacher and/or students will record the steps for the experiment.
- Teacher will also extend extra instruction to those struggling and will also re-shuffle the groups so that different ideas can be spread by different students into different groups.
- Summary
Students will have a step-by-step experiment to allow them to answer this question.

Student Exploration:

Stage 2

- **Small Group Work** – The students will perform the experiment and collect data. They will need to record their data on the provided worksheet. The next class period the groups should reconvene at computers to put the data into Excel and come up with their regression equation.
- **Whole Class Sharing/Discussion** – The small groups will then come back together to discuss what they came up with as a regression equation. The teacher should lead a group discussion on the reasons for differences in the equations. For best results, the teacher may want to combine all of the data from the whole class into one Excel spreadsheet and use them all to calculate a more accurate regression equation. This is an excellent time to discuss why more data gives you more accurate results.

Student/Teacher Actions:

- **What should students be doing?** The students should be conducting the experiment and then putting their data into Excel to determine their regression equations.
- **What should teachers be doing to facilitate learning?** Teachers should be constantly monitoring the groups to ensure they are on the right track. They should also be available for methodological questions but not hints on specific processes. If students are having trouble, they can use the textbook, internet, or classmates for help.
- **Possible questions** – Possible problems the students may face are getting started on the path for a proper experiment and determining multiple throws are needed.

Monitoring Student Responses

- Students will communicate with their peers in a group discussion their thinking.
- Students will conduct with a step-by-step experiment that will provide a correct answer.
- Teacher and/or students will record the results for the experiment.
- Teacher will also extend extra instruction to those struggling and will also re-shuffle the groups so that different ideas can be spread by different students into different groups.
- Summary
Students will conduct a step-by-step experiment and calculate a regression equation to allow them to answer this question.

Student Exploration:

Stage 3

- **Individual Work** – Each student should use the given information and the regression equation to determine from which balcony the stink bomb was thrown.
- **Small Group Work** – The students will come together in groups to discuss the individual calculations and to compare the calculations using their equation versus the teacher's equation.
- **Whole Class Sharing/Discussion** – The small groups will then come back together to discuss the answer to the problem. The teacher should lead a group discussion on the differences in each group's answer along with the comparison to the answer using the teacher's equation. Through group discussion and teacher input, the students should be able to determine which balcony the thrower was standing on.

Student/Teacher Actions:

- **What should students be doing?** The students should use their regression equations to calculate the answer.
- **What should teachers be doing to facilitate learning?** Teachers should be constantly monitoring the groups to ensure they are on the right track. They should also be available for methodological questions but not hints on specific processes. The teacher should also facilitate classroom discussion to guide students to the correct answer. If students are having trouble, they can use the textbook, internet, or classmates for help.
- **Possible questions** – Possible problems the students may face are plugging in the information properly and calculating correctly with the graphing calculator.

Monitoring Student Responses

- Students will communicate with their peers in a group discussion their thinking.
- Students will come up an answer using their equations.

- Teacher and/or students will record the answer to the problem.
- Teacher will also extend extra instruction to those struggling and will also re-shuffle the groups so that different ideas can be spread by different students into different groups.
- Summary
Students will have an answer to this question.

Assessment

- **Exit Pass will be called “Glad and Sad.” This will be handed to the student as a chart template on paper. The student will list what they would be “Glad” to see on a test and what they would be “Sad” to see on a test. This is not only what they did not like but also what they did not understand.**

Some ideas to think about when completing this chart:

- What did you like most about this lesson?
- What did you like least?
- What did you learn?
- What questions do you still have about the material covered in the lesson?

Homework Questions

PART 1– Will be included on a Student-Ready Worksheet at the end of the lesson.

Exit Passes will be evaluated to see what each student learned and if they still have any questions about the material covered that day. If the students still have questions about the material covered, they will be addressed at the beginning of class the next day.

- Homework will be graded as a homework grade. It will determine how the students comprehended the information learned in class.

Extensions and Connections (for all students)

- Lesson extensions/follow-up
- Connections to content in other subject areas. Examples of these would be included in the “Glad and Sad” portion of each lesson. Students will have to write complete sentences and organize ideas for this activity, therefore, it could be considered as grammatical work in English.

Strategies for Differentiation

The exit pass is developed so that students are able to express themselves by word or by picture. They can list or draw a picture of what they are glad or sad to see in the lesson. Another differentiation strategy would be to have larger groups or only worry about two floors. Gifted/ accelerated students can be asked to do further investigation. For example, the teacher could give them information on the heights of each student on the guilty balcony to determine which student threw the stink bomb.

Exit Pass – Homework 5

GLAD



SAD



Homework: Lesson 5

V. Write the quadratic equation for the given data.

x	4	6	8	10	12	16	18
y	52	113	196	306	441	782	989

1)

$$f(x) = \underline{\hspace{2cm}}$$

x	3	6	8	11	13	1	19
y	25	106	187	357	498	853	1066

2)

$$f(x) = \underline{\hspace{2cm}}$$

x	2	6	9	12	19	21
y	6	65	149	273	701	860

3)

$$f(x) = \underline{\hspace{2cm}}$$

VI. Predict the value of each of the above functions when $x = 23$.

4) $\underline{\hspace{2cm}}$

5) $\underline{\hspace{2cm}}$

6) $\underline{\hspace{2cm}}$

ANSWERS

Homework: Lesson 5

VII. Write the quadratic equation for the given data.

x	4	6	8	10	12	16	18
y	52	113	196	306	441	782	989

1)

$$f(x) = \underline{3.07x^2 - 0.59x + 5.26}$$

x	3	6	8	11	13	1	19
y	25	106	187	357	498	853	1066

2)

$$f(x) = \underline{2.96x^2 + .02x - 1.4}$$

x	2	6	9	12	19	21
y	6	65	149	273	701	860

3)

$$f(x) = \underline{2.03x^2 - 1.66x + 1.2}$$

VIII. Predict the value of each of the above functions when $x = 23$.

7) 1615.72

8) 1564.9

9) 1036.89

Data Collection Worksheet

[illegible]

Regression equation: _____

Lesson 6: Class is Going in Circles

Strand

Geometry

Mathematical Objective(s)

- The student will use measurement and properties of circles to arrange a classroom.
- The student will determine if a particular arrangement is even possible.

Mathematics Performance Expectation(s)

MPE 11: The student will use angles, arcs, chords, tangents, and secants to

- investigate, verify, and apply properties of circles;
- solve real-world problems involving properties of circles;

MPE 2: The student will collect and analyze data.

Related SOL

G.11 The student will use angles, arcs, chords, tangents, and secants to

- investigate, verify, and apply properties of circles;
- solve real-world problems involving properties of circles;

NCTM Standards

- Use mathematical models to represent and understand quantitative relationships
- analyze properties and determine attributes of two- and three-dimensional objects;
- draw reasonable conclusions about a situation being modeled.
- use visual tools such as networks to represent and solve problems;
- use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture

Materials/Resources

- Graphing Calculator
- Paper and Pencil
- Measuring tape or yard stick
- Formulas for measurements of a circle.

Assumption of Prior Knowledge

- Students should have a mastery of circumference and diameter of a circle.
- Students should have a mastery of measurement.
- Students should have a working knowledge of graphing calculators.
- Students should have a working knowledge of laying out furniture in a room on paper to measure feasibility.

Introduction: Setting Up the Mathematical Task

We have gotten an excellent reputation for using our mathematics to investigate and solve problems. Because of this, a teacher has come to us to have us solve a problem for her. The teacher wants to move the student desks to form a circle in her classroom, but doesn't know if it will work. She is pregnant so she can't move the desks over and over trying to see if it will work that way. She has come to us to have us determine how big the circle would have to be to fit all of her desks and if that circle would even fit in her classroom. If it doesn't fit, she would like suggestions for her classroom. (Ex. For a 20 ft by 24 ft classroom with 15 4ft wide tables would have a circumference of 60 ft and a diameter of 19.1 ft. This would fit in the classroom. However, if the tables were 5 ft wide, the circumference and diameter become 75 ft and 23.9 ft respectively which would not fit in the classroom.)

- Individual work (10 minutes), Small group discussion/work (60 minutes), whole class discussion (20 minutes)
- Student will work in groups of 2-3 on this activity.

Student Exploration:

- **Individual Work** – Each student should brainstorm to come up with an idea on how they can effectively answer the questions for the teacher.
- **Small Group Work** – The students will come together in groups to discuss the individual brainstorming and come up with a group idea stating the best way to come up with an answer. They will then need to measure the width of each desk and the size of the classroom as well as count the number of desks. With this information, they need to do calculations to determine if it will work. Sketches should be made to facilitate understanding. If the full circle will not fit in the classroom, the group should come up with an alternate plan for the arrangement of the classroom that will fit. Finally, the students should draw a final diagram of the classroom setup that they determined fit the classroom.
- **Whole Class Sharing/Discussion** – The small groups will then come back together to discuss what they came up with as a solution. The teacher should lead a group discussion on the results and conclusions from each group. If there are differences, the students should discuss what changes they made and why. Through group discussion and teacher input, the students should decide on the best arrangement for the classroom. One that is as close to the circle idea that the teacher had and fits the classroom. (It is up to the teacher if they want to choose a room that does or doesn't work or multiple rooms. It would also be a useful idea if the teacher chooses one of

each. If a teacher doesn't want to use actual classrooms, the measurements of an imaginary classroom and desk could be supplied.)

Student/Teacher Actions:

- **What should students be doing?** The students should be coming up with a classroom arrangement that will fit in the classroom trying to keep the original idea if possible.
- **What should teachers be doing to facilitate learning?** Teachers should be constantly monitoring the groups to ensure they are on the right track. They should also be available for methodological questions but not hints on specific processes. The teacher should also facilitate classroom discussion to guide students to the most appropriate classroom arrangement. If students are having trouble, they can use the textbook, internet, or classmates for help.
- **Possible questions** – Possible problems the students may face are using the correct formulas and processes to determine if the circle will fit and coming up with an alternate arrangement at first.

Monitoring Student Responses

- Students will communicate with their peers in a group discussion their thinking.
 - Students will determine the size of a circle layout that uses all of the desks and determine if it will fit in the given classroom.
 - Students will create a new arrangement for the teacher if the circle does not fit.
 - Teacher and/or students will record the final layout chosen by the students.
 - Teacher will also extend extra instruction to those struggling and will also re-shuffle the groups so that different ideas can be spread by different students into different groups.
- **Summary**
Students will have a classroom layout for the teacher .

Assessment

- **Exit Pass will be called "Glad and Sad." This will be handed to the student as a chart template on paper. The student will list what they would be "Glad" to see on a test and what they would be "Sad" to see on a test. This is not only what they did not like but also what they did not understand.**

Some ideas to think about when completing this chart:

- What did you like most about this lesson?
- What did you like least?
- What did you learn?
- What questions do you still have about the material covered in the lesson?

Exit Passes will be evaluated to see what each student learned and if they still have any questions about the material covered that day. If the students still have questions about the material covered, they will be addressed at the beginning of class the next day.

Extensions and Connections (for all students)

- Lesson extensions/follow-up
- Connections to content in other subject areas. Examples of these would be included in the “Glad and Sad” portion of each lesson. Students will have to write complete sentences and organize ideas for this activity, therefore, it could be considered as grammatical work in English.

Strategies for Differentiation

The exit pass is developed so that students are able to express themselves by word or by picture. They can list or draw a picture of what they are glad or sad to see in the lesson. Another differentiation strategy would be to have larger groups or only worry about two floors. Gifted/ accelerated students can be asked to do further investigation. For example, the teacher could give them other configurations to test in the classroom.

Exit Pass – Homework 6

GLAD



SAD



Worksheet for Measurements for the Classroom

1. Width of classroom _____
2. Length of classroom _____
3. Width of desks _____
4. Number of desks _____

Circumference Calculations: